

# Spatially-separated synchronised clocks in the same inertial frame: Time dilatation, but no relativity of simultaneity or length contraction

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## Abstract

The Lorentz transformation is used to analyse space and time coordinates corresponding to two spatially-separated clocks in the same inertial frame. The time dilatation effect is confirmed, but not ‘relativity of simultaneity’ or ‘relativistic length contraction’. How these latter, spurious, effects arise from misuse of the Lorentz transformation is also explained.

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The Lorentz transformation (LT) relates space-time coordinates  $(x, y, z, t)$  of an event in one inertial frame, S, to those of the same event  $(x', y', z', t')$  in another inertial frame S'. As is conventional, it is assumed that S' moves with uniform velocity,  $v$ , along the common  $x, x'$  axis of the two frames, and only events lying along the  $x, x'$  axis are considered.

In the following, it is assumed that the time  $t$  is registered by a clock at some fixed position in S. This records the same time, at any instant, as any synchronised clock at rest in S, at a different position, that may be compared locally with the reading of a moving clock. Two other clocks, C1 and C2, registering times  $t'_1$  and  $t'_2$  are at rest in S' on the  $x'$ -axis at  $x'_1 = 0$  and  $x'_2 = L'$  respectively. The equations of motion of these clocks in S are  $x_1 = vt_1$  and  $x_2 = vt_2 + L$  respectively, where  $L$  is the (time and velocity independent) separation in S of C1 and C2 any given instant  $t = t_1 = t_2$ .

The space-time LT for events on the world line of C1 is

$$x'_1 = \gamma[x_1 - vt_1] = 0 \quad (1)$$

$$t'_1 = \gamma[t_1 - \frac{vx_1}{c^2}] \quad (2)$$

while that for C2 is:

$$x'_2 - L' = \gamma[x_2 - L - vt_2] = 0 \quad (3)$$

$$t'_2 = \gamma[t_2 - \frac{v(x_2 - L)}{c^2}] \quad (4)$$

where  $\gamma \equiv 1/\sqrt{1 - (v/c)^2}$ .

Eqs. (1)-(4) are written in such a way that when  $t_1 = t_2 = t = 0$  then  $t'_1 = t'_2 = t' = 0$  so that, at this instant, C1 and C2 are synchronised. The constant  $L = x_2(t_2 = 0)$

corresponds to a particular ( $v$ -independent) choice of spatial coordinate system in the frame S. Since as  $v \rightarrow 0$ ,  $S \rightarrow S'$  and  $x \rightarrow x'$ , Eq. (3) becomes, when  $v = 0$

$$x'_2 - L' = x'_2 - L = 0 \quad (5)$$

from which it follows that  $L' = L$ . There is therefore no relativistic ‘length contraction’ (LC) effect.

Using (1) to eliminate  $x_1$  from (2) and (3) to eliminate  $x_2$  from (4) gives the time dilatation (TD) relations:

$$t_1 = \gamma t'_1 \quad (6)$$

$$t_2 = \gamma t'_2 \quad (7)$$

Therefore, at any given instant in the frame S when  $t_1 = t_2 = t$ , then  $t'_1 = t'_2$  and the clocks C1 and C2 remain synchronised —there is no relativity of simultaneity’ (RS) effect.

The TD relations (6) and (7), when  $t_1 = t_2 = t$ , may be compared to their Galilean limit where  $c \rightarrow \infty$ :  $t = t'_1 = t'_2$ . The TD relations are the only modifications of space time physics due the Lorentz transformation; the space transformation equations (1) and (3) become, when  $\gamma = 1$ , those of Galilean relativity, which give the same equations of motion  $x'_1 = 0$ ,  $x'_1 = vt_1$  and  $x'_2 = L$ ,  $x'_2 = vt_2 + L$ , of C1 and C2 respectively, in  $S'$ , S, as Eqs. (1) and (3).

As previously discussed in detail elsewhere [1, 2, 3], the spurious RS and LC effects of conventional special relativity are the result of an incorrect use of the space-time LT to analyse space and time measurements. For example, the coordinates of C2 are simply substituted into the LT (1) and (2) appropriate for the clock C1 giving:

$$x'_2 = L' = \gamma[x_2 - vt_2] \quad (8)$$

$$t'_2 = \gamma[t_2 - \frac{vx_2}{c^2}] \quad (9)$$

Setting  $t_2 = 0$  in (8) gives

$$L' = \gamma x_2(t_2 = 0) = \gamma L \quad (10)$$

while using (8) to eliminate  $x_2$  from (9) gives:

$$t'_2 = \frac{t_2}{\gamma} - \frac{\gamma v L}{c^2} \quad (11)$$

in contradiction to the translationally-invariant TD relation of Eq. (7). Eqs. (10) and (11) are typical of text-book ‘derivations’ of the LC and RS effects respectively. The former was given in Einstein’s original special relativity paper [4].

The LT (3) and (4) with  $L' = L$  may be written as:

$$x'_2 = \gamma[x_2 - vt_2] + X(L) \quad (12)$$

$$t'_2 = \gamma[t_2 - \frac{vx_2}{c^2}] + T(L) \quad (13)$$

where  $X(L)$  and  $T(L)$  are the constants:

$$X(L) \equiv (1 - \gamma)L \quad (14)$$

$$T(L) \equiv \frac{\gamma v L}{c^2} \quad (15)$$

The necessity to add such constants to the right sides of (1) and (2) in order to correctly describe synchronised clocks at different spatial positions was already pointed out by Einstein, though, to the present writer's best knowledge, this was never done, either by Einstein himself or other authors, before the work presented in Ref [1]. The important passage occurs in §3 of Ref [4], immediately after the derivation of the LT as in (1) and (2) above. In the English translation of Perrett and Jeffery it is: 'If no assumption is made as to the initial position of the moving system and as to the zero point of  $\tau$ ' ( $\tau$  is  $t'$  in the notation of the present letter) 'an additional constant is to be placed on the right side of each of these equations.' ('these equations' are equivalent to Eqs. (1) and (2) of the present letter).

The essential point made here is that physics, either that underlying the clock mechanism, or the relativistic TD effect of Eqs. (6) and (7), predicts only the observed *rate* of a moving clock, not its setting. The spurious RS and LC effects arise because a clock *setting*, built into the 'standard' LT (1) and (2), is misinterpreted as a physical *time difference* between the readings of synchronised clocks at a different spatial positions in an inertial frame, when they are viewed by an observer in a different inertial frame. .

Experiments have recently been proposed to search for the existence (or not) of the RS effect [5]. At the time of writing, there is ample experimental verification of TD but none of RS or LC [1].

## References

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